GeoMechanica
Geotechnical Engineering Services

PyDy

Theory and Engineering Manual

Contents

Pile Dynamica: A suite of software for linear dynamic analysis of pile foundations

Pile dynamica or PyDy consists of 4 modules for the analysis of pile subjected to dynamc loads. The modules are VDAP: for vertically loaded piles, LDAP: for laterally loaded piles, RDAP: for piles subjected to moments, and TDAP: for piles under torsion. This manual will explain the theory behind PyDy and illustrates how the program can be used to analyze piles subjected to dynamic loads.

1. Theory Manual

The program uses a soil-pile interaction model where the pile is modeled as either a vertical bar element (for vertical and torsional analysis) or a beam element (for Lateral and rocking analysis). The soil is modeled as viscoelastic oscillator with spring and damper. The spring represent soil stiffness related to the direction of the load and the damper represent both radiation and material damping of the soil. The method is successfully used in pile driving analysis see [1] and for seismic analysis See [2]. In this suite of programs, the method is extended to all directions of loading and is intended for the analysis of pile foundations under any dynamic loading condition. Dynamic loading includes vibrations resulting from machines, seismic events, winds or waves. The programs use the finite element method to solve the equation of motion of the problem in time domain. The output shows variation of deformation with time at the level of the pile head, which can be used to tweak the performance of the pile to meet certain criteria.

1.1. Time history analysis

If the engineer is interested in the time history analysis of the pile under certain time dependent loads (e.g. Sinusoidal, impact etc.), an option is provided to obtain the displacement at the pile head during analysis time and along the pile length at a certain time step. The program uses a trapezoidal integration scheme (average acceleration method) to obtain displacement of the pile head as a function of time.

1.2. Discretization of the pile and the soil

In this suite of programs, the pile is divided into bar elements for the purpose of analyzing piles that are subject to vertical and torsional loads. The pile is divided into beam elements to analyze piles that are subject to lateral and rocking loads. The discretization schemes are shown in figure 1.

For a bar element used to analyze vertical pile response the stiffness matrix of a single pile segment is obtained as

$$Pile\ stiffness\ matrix\ [k_p] = \frac{E_p\,A}{L/n}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \tag{2}$$

Where E_p Is the pile young's modulus, A is the pile cross-sectional area, L is the pile length and n is the segments the pile is divided into.

For torsional analysis the stiffness matrix of a single pile segment is

$$Pile\ stiffness\ matrix\ [k_p] = \frac{G_p\, J}{L/n}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \tag{3}$$

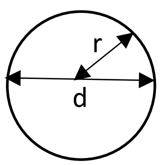
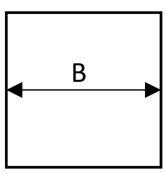
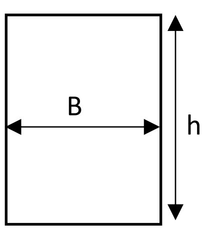
Where G_p is the shear modulus of the pile and J is the torsional constant which depends on the shape of the pile cross-sectional area. For lateral and rotational analysis, the pile is modeled as a beam element and the stiffness matrix of one segment of the pile is 4 by 4 matrix:

$$Pile\ stiffness\ matrix\ [k_p] = \frac{E_p\, I}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6\mathrm{L} \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \tag{4}$$

Where I is the second moment of inertia of the pile section. Table 1 shows equations for the area, torsional constant and area moment of inertia. These constants define the shape of the pile and are needed as an input by the user.

The stiffness matrices shown in equations 2,3 and 4 are for one segment of the pile and are assembled into a global stiffness matrix which size is dependent on the number of segments of the pile. The mass matrix of one segment of the pile is 2 by2 for vertical and torsional analysis where entries [1,1] and [2,2] are equal to $1/2\ \rho\ A\ \mathrm{L/n}$. For lateral and rocking analysis the pile mass matrix is a 4 by 4 matrix where entries [1,1] and [3,3] of the mass matrix are equal to $1/2\ \rho\ A\ \mathrm{L/n}$. Where ρ is the mass density of the pile. It is assumed that the pile material damping is very small. The pile damping matrix is a zero matrix.

Table 1: Shape constants for different shapes

Shape	Area	Torsion Constant, J	Second moment of inertia, I
	πr^2	$\dfrac{\pi r^4}{2}$	$\dfrac{\pi r^4}{4}$
	B^2	$0.141B^4$	$\dfrac{B^4}{12}$
	bh	$bh^3\left(\dfrac{1}{3}-\dfrac{0.21B}{h}\left(1-\dfrac{B^4}{12h^4}\right)\right)$	$I=\dfrac{bh^3}{12}$ Or $I=\dfrac{hb^3}{12}$ Depending on the direction of the bending moment.

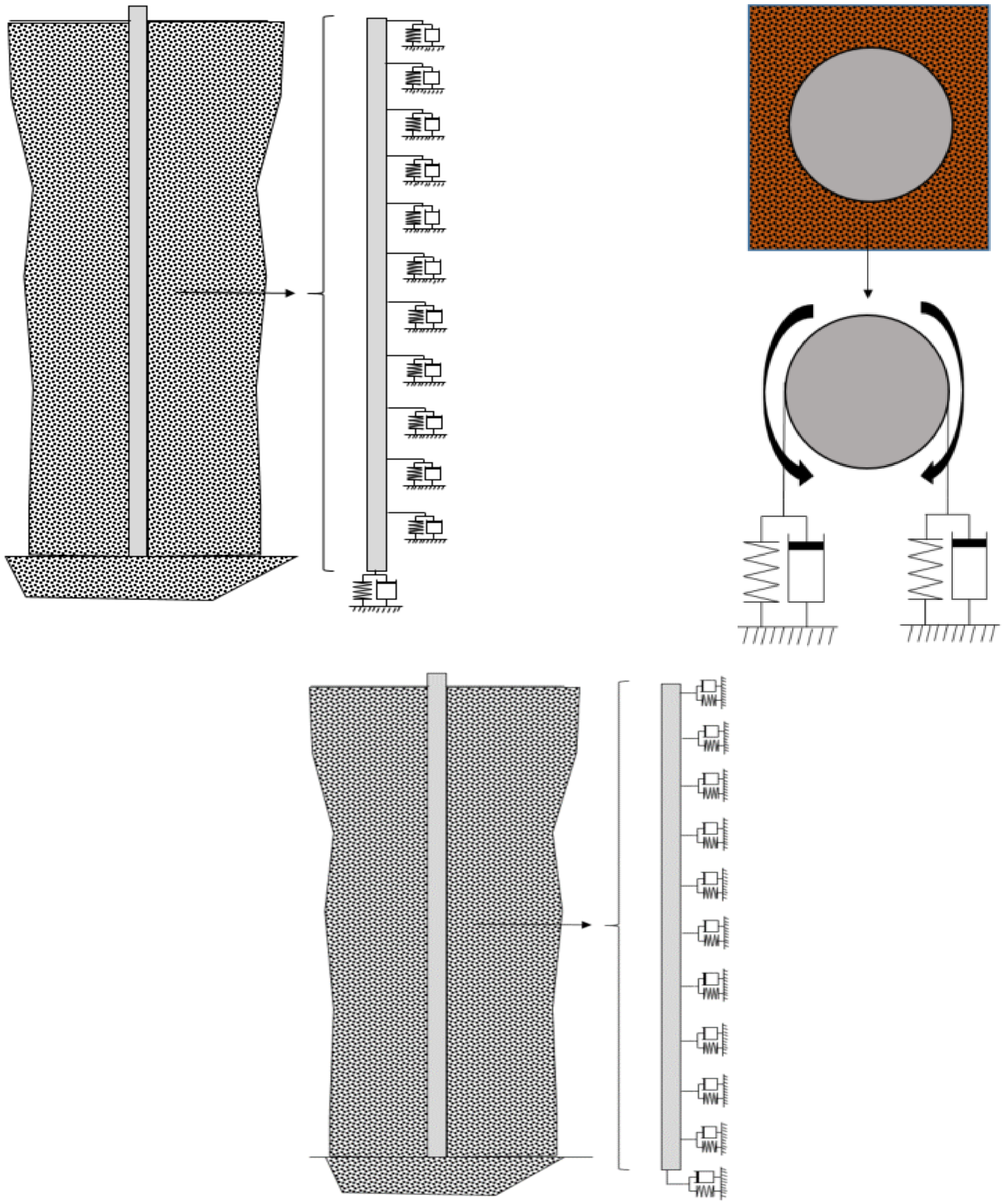
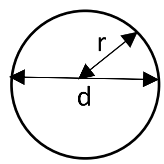
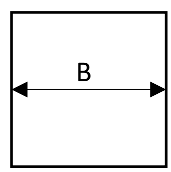
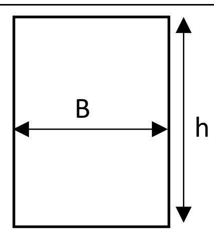


Figure 1: shows models used to discretize the pile. Top left: model used for vertically loaded piles. Top right: top view of model used for torsional analysis of piles. Bottom: model used for lateral and rocking analysis of piles.

For piles that undergoes torsional vibration a specific input of density moment of inertia is needed to be input instead of the density of the pile. The density moment of inertia (unit = $kg\ m^2\ /m^3$) of different shapes is shown in table 2. The program then multiplies this value by the volume of one pile segment to obtain the mass moment of inertia and place it in the mass matrix instead of the mass of a pile segment)

Table 2: density moment of inertia for different shapes

Shape	Density second moment of inertia for use in torsional analysis
	$\frac{\rho r^2}{2}$
	$\frac{\rho}{12} B^2$
	$\frac{\rho}{12} h^2$

1.3. Modeling of the soil

The soil as shown in figure 1 is modeled using springs and dampers and each set of spring and damper is attached to one segment of the pile. At the bottom the soil is also modeled as a set of spring and damper that is attached to the base of the pile. The values of the spring stiffness and the damper coefficient differ depending on the analysis type, properties of the soil layers. Several equations are provided in literature for these values.

1.3.1. Recommended equations for calculation of spring stiffness and damping for the soil at the side and soil at the base

The following is a review of values for the spring stiffness and damping for the soil at the side of the pile and at the base. Detailed equations for each type of analysis are presented that can be used to calculate the soil reaction and plug it in the program. Examples of the calculation is presented in the engineering section of this manual.

1.3.1.1. Side soil spring stiffness & damping for vertical load

For vertical vibration of the pile, the side soil will provide frictional resistance and damping for the waves that will propagate from the pile to the soil through shear. The springs stiffness for a layer of soil at the side of the pile is calculated according to Randolph & Simmons 1986 [3] as

$$k_s = \delta G_s \frac{N}{m^2} per\ unit\ length\ of\ pile \tag{5}$$

Where δ `is calculated as

$$\delta = \frac{2\pi}{\text{Ln}\left(\frac{2r_m}{d}\right)} \tag{6}$$

Where d is the diameter of a circular pile, width of a square pile or the shortest dimension of a rectangular pile. r_m Is calculated as

$$r_m = \chi_1 \chi_2\, L\,(1 - \mu_s) \tag{7}$$

Where μ_s Is the soil Poisson's ratio. χ_1And χ_2 Are factors to account for soil inhomogeneity. $X_1 = 2.5$ for friction piles and $\chi_1 = 2$ for end-bearing piles or if the bed rock is at a depth $\leq 2.5L$. For friction piles $\chi_2 = 1$ and for end-bearing piles $\chi_2 = \ 0.5$. Alternatively Gazetas & Mylonakis 1998 [4] suggest that $\chi_1 \chi_2 = 2.5$ for friction piles and $\chi_1 \chi_2 = 1$ for bedrock conditions.

Randolph & Simmons 1986 suggested the radiation damping of the side soil to be

$$c_s = \frac{G_s}{v_s}\ N\ s/m^3 \tag{8}$$

Where G_s Is the soil shear modulus and v_s Is the soil shear wave velocity.

Randolph and Simmons also suggested a spring stiffness of the side soil to be calculated as

$$k_s = \frac{1.375 G_s}{\pi r} \frac{N}{m^3} \tag{9}$$

According to Gazetas 1991 [5] the side damping of an irregularly shaped embedded foundation can be taken as

$$c_s = \rho_s v_s A_{side}\ N\ s/m \tag{10}$$

Where ρ_s Is the soil mass density, v_s Is the soil shear modulus and A_{side} Is the side area of the pile shaft segment that is in contact with the soil.

1.3.1.2. Base soil spring stiffness and damping for vertical load

For the base of the pile Lysmer & Richart 1966 model can be implemented and the stiffness at the base is

$$k_b = \frac{4G_s r}{1-\mu_s}\ N/\ m \tag{11}$$

While damping can be calculated as

$$c_b = \frac{3.4r^2}{1-\mu_s}\sqrt{\rho_s\ G_s}\ N\ s/m \tag{12}$$

1.3.1.3. Side soil spring stiffness & damping for lateral & bending loads

Although lateral & bending loading are analyzed separately, the modeling is the same for both analyses. Th pile is modeled as a beam in both analyses. The soil is assumed to resist only lateral loads, since soil can't take rotation. Any bending of the pile will result in lateral deformation of the soil even if there is no lateral load applied. In addition, spring stiffness derived for laterally loaded piles consider the bending capability of the pile. the last two sentences suggest that there is no need to use a different value for the spring stiffness of laterally loaded piles if the pile is modeled as a beam. For the soil at the side of a laterally loaded pile the spring stiffness can be calculated according to Vesic 1961 [6] as

$$k_s.B = \frac{1.3(E_s)}{(1-\mu_s^2)}\sqrt[12]{\frac{E_s B^4}{E_p I}}\ N/m^3 \tag{13}$$

Where E_s is the modulus of elasticity of the soil, B is the diameter of a circular pile, width of a square pile, or shortest dimension of a rectangular pile, μ_s is Poisson's ratio of the soil, E_p is the pile modulus of elasticity and I is the moment of inertia of the pile. Note that the spring stiffness calculated using equation 13 needs to be multiplied by the surface area of the pile segment and divided by the B. Equation 13 is developed for beams on elastic foundations in general. Reese (INSERT REFERENCE) suggested using the elastic theory in calculation of the spring stiffness as

$$k_s = \frac{E_s}{B(1-\mu_s^2)}\ \ N/m^3 \tag{14}$$

Here, equation 14 needs to be multiplied by the half side area of the pile segment. B is the diameter of a circular pile or the width of rectangular or square piles.

Damping of the side soil can be calculated using equations suggested by Gazetas 1991 [5]. According to this method the damping of the side soil for any shape of rigid foundation embedded in an elastic soil is calculated and is modified in the following equation **for a circular pile**

$$c_s = 4\ \rho_s\ v_s\, r\ L_{seg} +\ 4\ \rho_s\ v_{LA}\ r\ L_{seg} \tag{15}$$

Note that equation 15 is a modified version of the original equation to be suitable **for a circular pile foundation. For a rectangular foundation** equation 15 becomes

$$c_s = 4\,\rho_s\,v_s\,\frac{B}{2}\,L_{seg} + 4\,\rho_s\,v_{LA}\frac{h}{2}\,L_{seg} \tag{16}$$

Where ρ_s is the soil density, v_s is the soil shear wave velocity, r is the radius of the pile, h is the length of a rectangular pile cross section, B is the width of a rectangular pile and v_{La} is the lysmer analog soil wave velcotiy and is calculated as

$$v_{LA} = 3.4/(\pi(1-\mu))\;v_s \tag{17}$$

Note that the term $B/2$ and $h/2$ could switched depending on the direction of the loading. The presented arrangement in equation 16 is for a loading that is parallel to the width B of the foundation.

1.3.1.4. Base soil spring stiffness & damping for lateral & bending loads

For the base soil, the stiffness the soil at the pile base can be calculated from theory of elastic half space as shown by Bycroft and Wartburton 1955 [7]

$$k_b = \frac{32(1-\mu)G_s r}{7-8\mu} \tag{18}$$

And The damping is

$$c_b = \frac{18.4(1-\mu)}{7-8\mu} r^2\sqrt{\rho_s G_s} \tag{19}$$

1.3.1.5. Side soil spring stiffness & damping for torsional loading

The stiffness of the side of the pile resisting a torsional dynamic loading is provided by Gazetas 1991 as

$$k_s = k_{t,base}\,\Gamma_w\,\Gamma_{tre} \tag{20}$$

Where $k_{t,base}$ is torsional stiffness at the base of the pile segment and is calculated as

$$k_{t,base} = 3.5\,G_s I_{bz}^{0.75}\left(\frac{B}{h}\right)^{0.4}\left(\frac{I_{Bz}}{\left(\frac{B}{2}\right)^4}\right)^{0.2} \tag{21}$$

Γ_w is a factor and is calculated as

$$\Gamma_w = 1 + 0.4\frac{j_s}{j_r}\left(\frac{B}{2L_{seg}}\right)^{0.6} \tag{22}$$

Γ_{ue} is another factor calculated as

$$\Gamma_{tre} = 1 + 0.5\left(\frac{2L_{seg}}{B}\right)^{0.1}\left(\frac{(B/2)^4}{I_{bz}}\right)^{0.13} \tag{23}$$

j_s is calculated as

$$j_s = \frac{4}{3}L_{seg}\left(\left(\frac{B}{2}\right)^3 + \left(\frac{h}{2}\right)\right)^3 + BhL_{seg}(\frac{h}{2} + \frac{B}{2}) \tag{24}$$

j_r is calculated as

$$j_r = \frac{4}{3}Bh(\frac{B^2}{4} + \frac{h^2}{4}) \tag{25}$$

And I_{bz} is the moment of inertia about the pile vertical axis. note that equation 20 is the static torsional stiffness and is valid for many cases as will be shown later. The dynamic stiffness is a function of the frequency the side resistance might be negligible. This evident from Gazetas 1991 stating that and embedded footing has the same torsional stiffness as a footing on the surface. Although seems inaccurate since torsional loading of a foundation that is embedded will develop shear along the soil-foundation interface.

The damping provided by Gazetas 1991 is frequency dependent and is calculated as

$$c_s = 4\rho_s L_{seg}\left[\frac{1}{3}v_{La}\left(\frac{h^3}{8} + \frac{B^3}{8}\right) + v_s\frac{Bh}{4}\left(\frac{h}{2} + \frac{B}{2}\right)\right]\eta_t \tag{26}$$

Where η_t is a function of the frequency and is calculated as

$$\eta_t = \frac{a_0^2}{\left[a_0^2 + \frac{1}{2}\left(\frac{h}{B}\right)^{-1.5}\right]} \tag{27}$$

Where a_0 is called dimensionless frequency and is calculated as

$$a_0 = \frac{\omega\, r}{v_s} = \frac{2\,\pi\, f\, r}{v_s} \tag{28}$$

Where ω is the frequency in radians per second and f is the frequency in Hertz.

1.3.1.6. Base spring and damper coefficient for torsional loading

For torsional dynamic loading the stiffness at the base of the pile can be calculated using equation 21 or using the following equation

$$k_b = \frac{16}{3} G_s r^3 \tag{29}$$

Damping of the base against torsional loading is

$$c_b = \frac{0.5}{1+2B_\alpha} \left(2\sqrt{k_b\, J_m}\right) \tag{30}$$

Where J_m is mass moment of inertia and is calculated for circular piles as

$$\frac{1}{2}\left(\pi r^3 * h * \frac{\gamma}{g}\right) \tag{31}$$

Where γ is the unit weight of the base element, h is its thickness, r is the radius, g is the gravity acceleration and B_α is calculated as $j/\rho_s r^5$.

1.3.2. Remarks on equations for calculation of side and base stiffness and damping

The equation presented earlier are obtained using recommendations of the literature. Some base equations are defined for the vibration on the surface of elastic half-space but can be used at the base as an approximation for vertical loading.

The stiffness and damping provided are independent of the frequency for all the cases. The stiffness provided was taken from static stiffness for all cases except for torsional loading where a frequency dependent stiffness is given.

The stiffness and damping equations are given as an approximation and the user can use their own equations or use those recommended by literature in calculating the stiffness and damping.

Damping given in these equations is related to geometrical damping of the soil. Geometrical damping occurs as a result of the waves moving away from the source of vibration. The damping provided in the equations will simulate the infinite medium of the soil. Material damping which occurs as a result of the material ability to absorb deformation is not given. The user can assume a 5 % material damping of the stiffness of the soil or any value calculated from proper soil testing.

For use of equations that are specified for rectangular and square (equations that take foundation width) or equations that are specified for circular foundations (equations that only take radius of foundation as an input), use equation 31 which calculates the equivalent radius of a foundation from a rectangular or square foundation.

$$r = \sqrt[4]{\frac{BL^3}{3\pi}} \tag{32}$$

2. Engineering Manual

The engineering part of the guide is a practical guide that shows how to implement the theory and use the software for the calculation of the pile response for different modes of vibration. example problem is given. The problem is solved to obtain the pile response to different modes of loading and at different frequencies. The user will be able to use the example as guidance in implementing the software.

2.1. Example Problem: Pile under vertical and horizontal dynamic loading in homogeneous soil with comparison against theoretical models and finite element method

A pile which is 30 meters in length and 1 meter in diameter with a modulus of elasticity of $2.1x10^{10}$ N/m^2 is embedded in a soil which has a modulus of elasticity of $2.1x10^8$ N/m^2 and a Poisson's ratio of 0.4. The density of the soil is about 1835 kg/m^3. The pile and the soil are shown in figure 2.

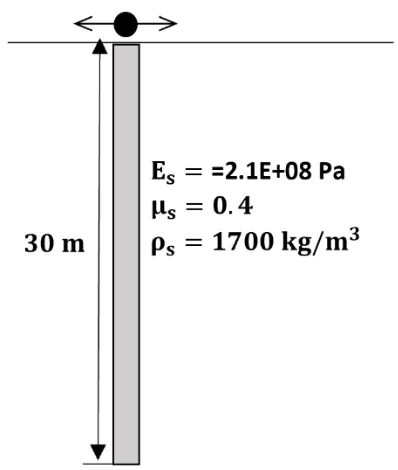


Figure 2: showing the pile and the soil.

1. For vertical dynamic loads:

The stiffness of the soil springs and the dashpot damping are calculated using equations 5 and 10 respectively. these equations are for the soil along the pile length. At the base of the pile the stiffness and damping are calculated using equations 11 and 12 respectively. A sinusoidal vertical dynamic force is applied to the pile with multiple frequencies. the piles was analyzed using **VDAP**. The input of the program is shown in figure 3 and 4 while a sample window showing the program results is shown in figure 5. A comparison of the calculated u_d/u_s at different frequencies using PyDy and 3D finite element method is shown in figure 6. The repsonse of the pie can be captured accurately using the PyDy and much faster compared to the rigorous finite element modelling. Note that accuracy of the program depends on the choice of the soil spring stiffness and damping. much more accurate equaitons can be found ijn literature.

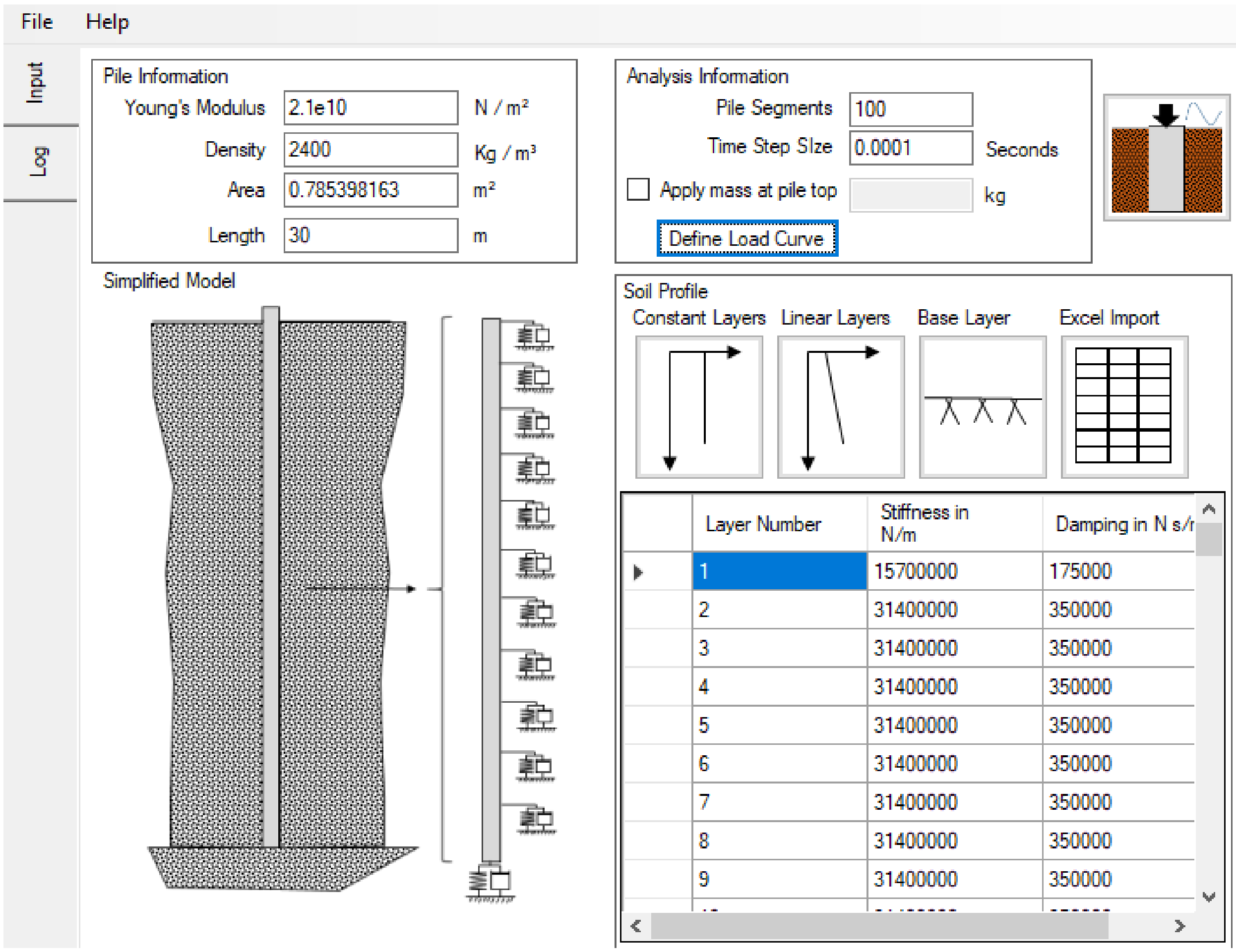


Figure 3: Main VDAP window with input of soil and pile data.

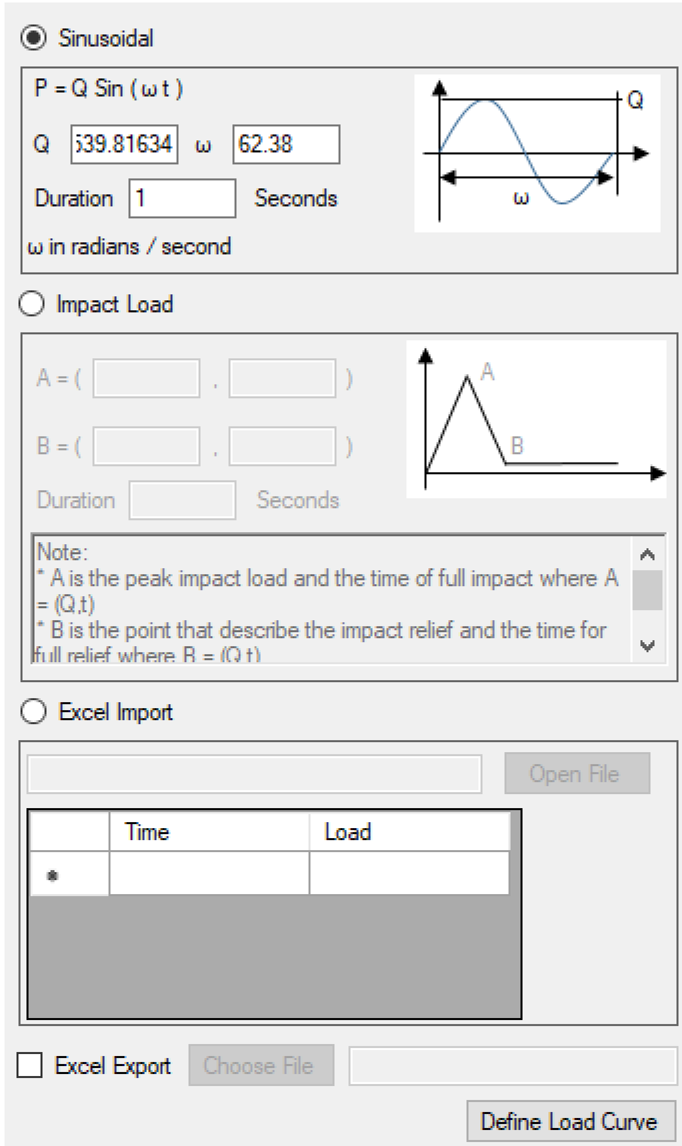


Figure 4: Load input window of VDAP.

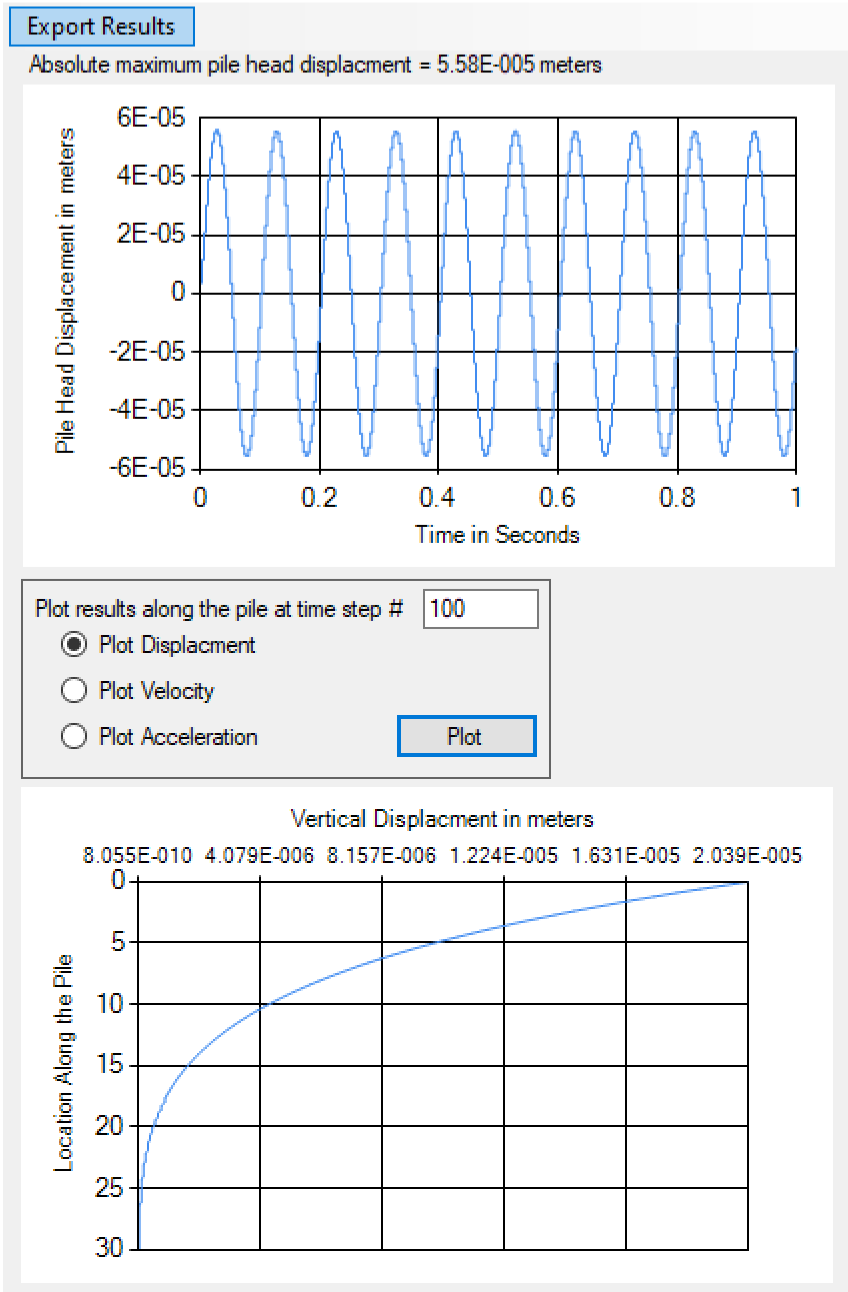


Figure 5: Results of VDAP at 10 Hz.

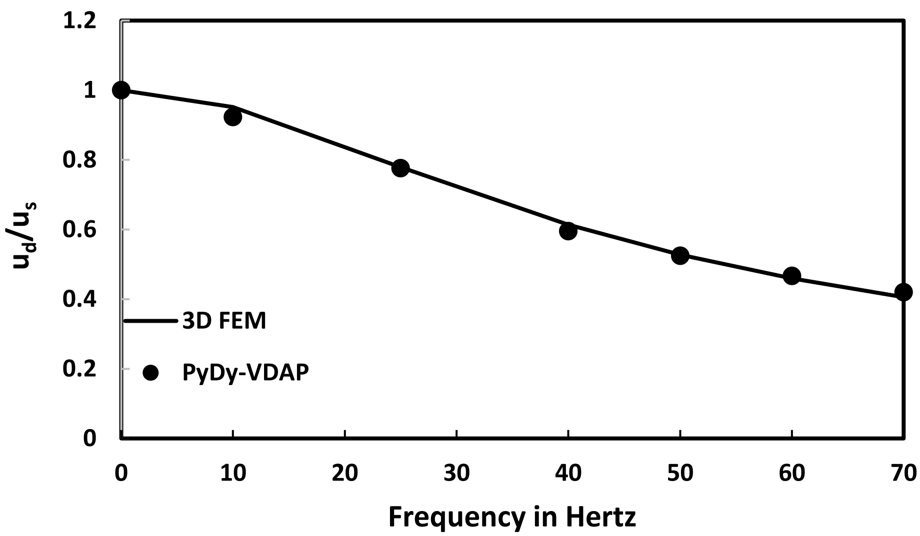


Figure 6: u_d/u_s calculated using VDAP compared to 3D FEM.

2. For lateral loading

The stiffness of the soil springs and the dashpot damping are calculated using equations 14 and 15 respectively. These equations are for the soil along the pile length. At the base of the pile the stiffness and damping are calculated using equations 18 and 19 respectively. A sinusoidal lateral dynamic force is applied to the pile with multiple frequencies. The piles was analyzed using **LDAP**. The input of the program is shown in figure 7 and 8, while a sample of the program output at 10 Hz is shown in figure 9. A comparison of the calculated u_d/u_s at different frequencies using PyDy and finite element method is shown in figure 10. The response of the pie can be captured reasonably well using the PyDy and much faster compared to the rigorous finite element modelling. Note that accuracy of the program depends on the choice of the soil spring stiffness and damping. much more accurate equaitons can be found ijn literature. . Note that accuracy of the program depends on the choice of the soil spring stiffness and damping. much more accurate equaitons can be found ijn literature.

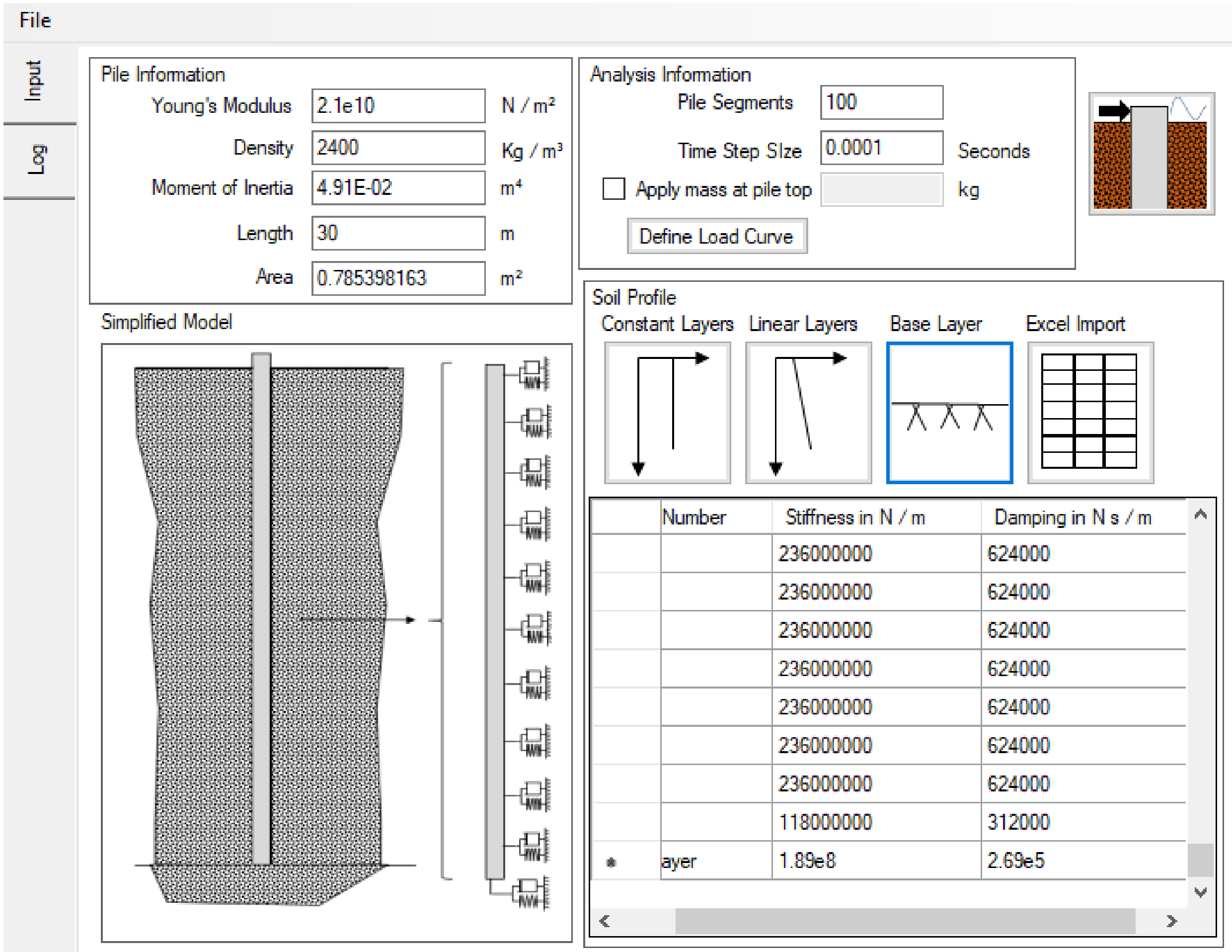


Figure 7: Showing main LDAP window with input of soil and pile data.

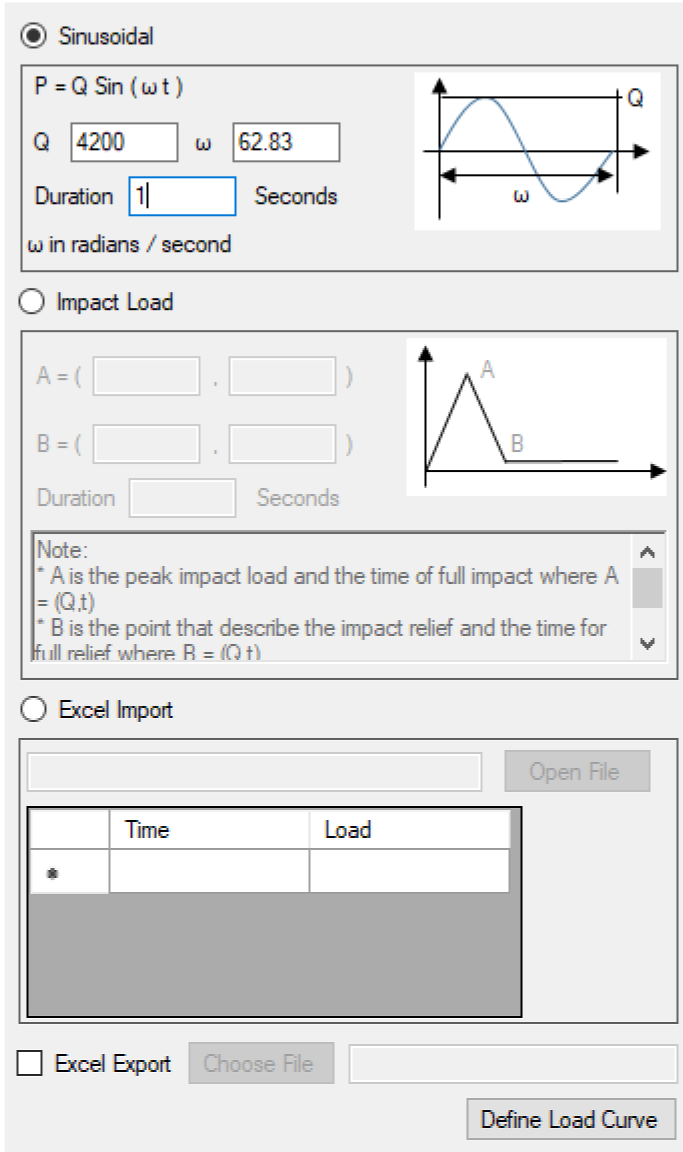


Figure 8: Load input window of LDAP.

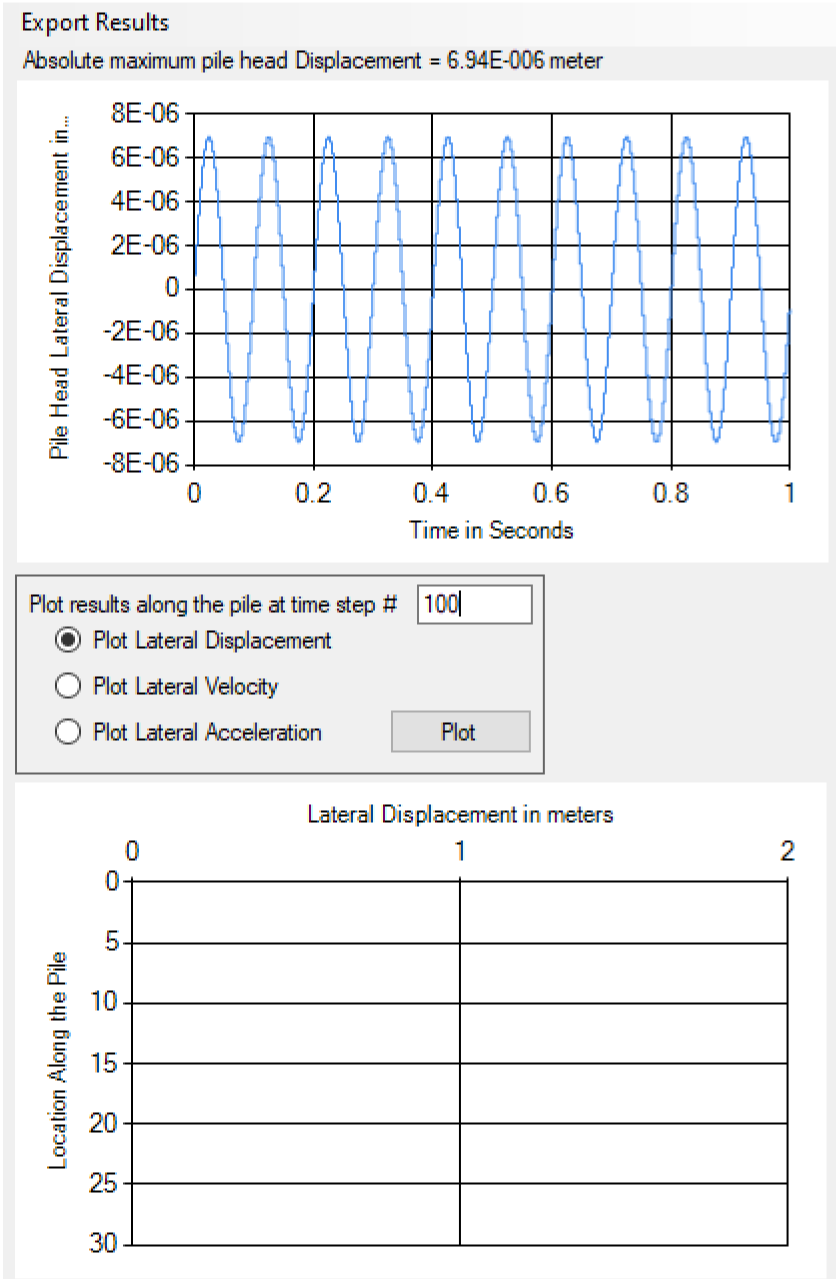


Figure 9: Results of LDAP at 10 Hz.

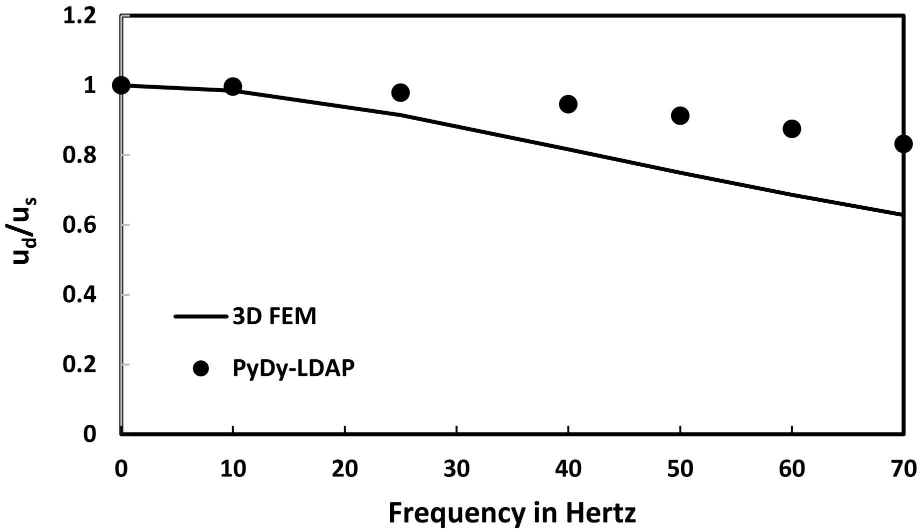


Figure 10: u_d/u_s calculated using LDAP compared to 3D FEM.

References

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[2] S. Prakash, *Soil Dynamics*. Mcgraw-Hill, 1981.

[3] M. F. Randolph and P. Wroth, "Analysis of Deformation of Vertically Loaded Piles," *J. Geotech. Eng. Div.*, vol. 104, no. 12, pp. 1465–1488, 1978.

[4] G. Gazetas and G. Mylonakis, "Settlement and additional internal forces of grouped piles in layered soil," *Géotechnique*, vol. 48, no. 1, pp. 55–72, 1998.

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[9] A. M. Kaynia and E. Kausel, "Dynamics of piles and pile groups in layered soil media," *Soil Dyn. Earthq. Eng.*, vol. 10, no. 8, pp. 386–401, 1991.